Session 7

Exercise 1

Using the scaling hypothesis for the free energy and and the form of the correlation function of the Ising model, prove that the thermodynamic inequalities for the critical exponents are equalities

Rushbrooke inequality: $\alpha + 2\beta + \gamma \geq 2$

Griffiths inequality: $\alpha + \beta(1+\delta) \geq 2$

Josephson (hyperscaling) relation: $2 - \alpha \geq d\nu$

(because it involves d)

Fisher's inequality: $\gamma \geq (2-\eta)\nu$

Remembering that $\left(\frac{T_c - T}{T_c} = t \right)$

magnetization $m \sim t^{\beta}$ $(t > 0)$

specific heat $C \sim |t|^{-\alpha}$

$m \sim h^{1/\delta}$ $(t = 0)$

susceptibility $\chi \sim |t|^{-\gamma}$